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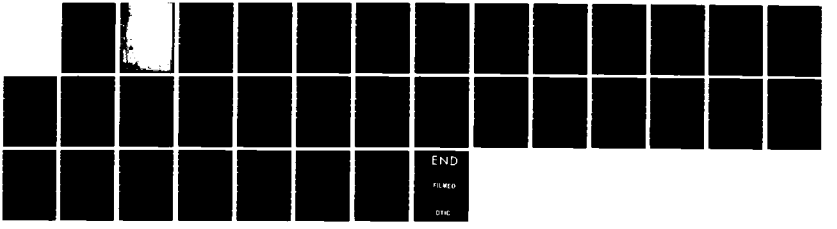

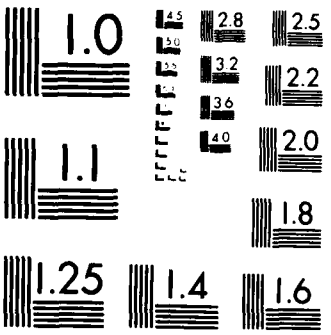

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Extensive tables of formulae for the orbital eccentricity function $K_{\ell pq}(e)$ have been generated using the MACSYMA* symbolic algebraic manipulation system. These formulae are presented through tenth order in the orbital eccentricity for $2 \leqslant \ell \leqslant 10$, $0 \leqslant p \leqslant 5$, and $-2 \leqslant q \leqslant 2$. Comparisons are made with tabulations obtained by other workers.

*MACSYMA is a large symbolic manipulation program developed at MIT and supported during 1975-1983 by NASA, ONR, DOE, and the USAF, and since 1982 by Symbolics, Inc. of Cambridge, Mass. MACSYMA is a trademark of Symbolics, Inc.

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FOREWORD

The development of symbolic algebraic manipulation systems for use on the digital computer is a fairly recent technical innovation that has not even begun to be fully exploited. The potential for application of these systems to the nonnumeric resolution of complex problems seems boundless. The purpose of this report is to provide the results obtained from the utilization of one such "state of the art" system, MACSYMA, to provide high-order expansion formulae for the classical orbital eccentricity function found in celestial mechanics theory. These formulae are generally applicable to low eccentricity orbits, and the relative ease with which they were generated demonstrates MACSYMA's analytical prowess. This report was reviewed by Dr. R. J. Anderle and Mr. R. W. Hill.

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THOMAS A. CLARE, Head
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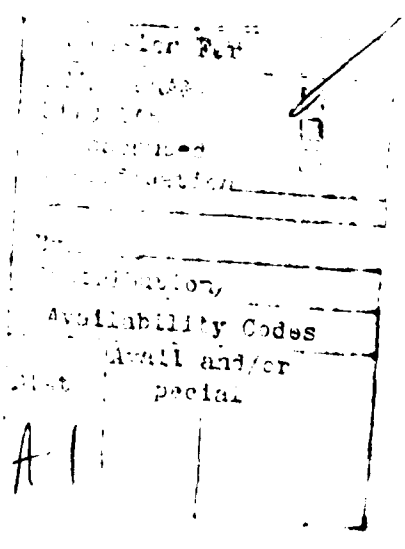

CONTENTS

INTRODUCTION

Although digital computers have been performing complex numerical computations for several decades, their application to the solution of nonnumeric problems is a relatively recent development.[1,2] Several sophisticated symbolic computer algebra systems currently exist and can serve as a most valuable asset to the scientific researcher.[3,4] Such systems can provide general solutions to a wide variety of problems in a manner that eliminates the need for tedious, manually performed algebraic operations, while simultaneously minimizing the likelihood of introducing errors into the solution.

One such "state of the art" system is the MACSYMA computer algebra system developed by the Mathlab group at the Massachusetts Institute of Technology.[5] In this work, MACSYMA has been used to generate expansion formulae for the eccentricity function through tenth order in the orbital eccentricity. These formulae are presented in tabular form for $2 \leqslant \ell \leqslant 10$, $0 \leqslant p \leqslant 5$, and $|q| \leqslant 2$ and may be used to verify numerically generated values for $K_{\ell pq}(e)$ or applied directly to the analytical resolution of orbital dynamics problems. For the sake of completeness, the following sections also include a mathematical description of the function $K_{\ell pq}(e)$ as well as a comparison of results with those obtained by other workers.

THE $K_{\ell pq}(e)$ ECCENTRICITY FUNCTION

The $K_{\ell pq}(e)$ eccentricity function is related to the $G_{\ell pq}(e)$ function used by Kaula[6] through the expression

$$G_{\ell pq}(e) = e^{|q|} K_{\ell pq}(e) \tag{1}$$

where,[7] for $q > 0$:

$$K_{\ell p q}(e)=(-1)^{|q|}\,2^{\ell}(1+\gamma)^{-\ell-|q|}\sum_{k=0}^{\infty}\sum_{r=0}^{|q|+k}\sum_{t=0}^{k}\frac{(-1)^r}{r!t!}\binom{2p-2\ell}{|q|+k-r}\binom{-2p}{k-t}\left(\frac{\ell-2p+q}{2}\right)^{r+t}(1+\gamma)^{r+t-k}(1-\gamma)^{k} \tag{2}$$

for $q < 0$:

$$K_{\ell p q}(e) = (-1)^{|q|}\,2^{\ell}(1+\gamma)^{-\ell-|q|}\sum_{k=0}^{\infty}\sum_{r=0}^{|q|+k}\sum_{t=0}^{k}\frac{(-1)^t}{r!t!}\binom{-2p}{|q|+k-r}\binom{2p-2\ell}{k-t}\left(\frac{\ell-2p+q}{2}\right)^{r+t}(1+\gamma)^{r+t-k}(1-\gamma)^{k} \tag{3}$$

and for $q = 2p - \ell$:

$$K_{\ell p(2p-\ell)}(e) = \gamma^{-2k+1}\sum_{k=0}^{p'-1}\binom{\ell-1}{2k+|2p-\ell|}\binom{2k+|2p-\ell|}{k}(1-\gamma^2)^{k}\,2^{-2k-|2p-\ell|} \tag{4}$$

where

$$\gamma = \sqrt{1-e^2} \tag{5}$$

and

$$p' = \frac{\ell-|2p-\ell|}{2} \tag{6}$$

DISCUSSION

Equations 2 through 4 of the previous section were coded in the MACSYMA programming language and processed by the MACSYMA system to generate the tables of expansion formulae provided in the attached appendix. Since the majority of satellite orbits to which these results might be applied are of low eccentricity, it was found to be adequate to set the upper limit for the summation index k in Equations 2 and 3 to 5 and constrain the q subscript to $|q| \leq 2$.[8]

The MACSYMA generated results were compared with those obtained by other workers and agreed identically with those fourth-order formulae presented by Giacaglia[7] for $2 \leq \ell \leq 4$, $0 \leq p \leq 4$, and $|q| \leq 2$. The MACSYMA formulae also compared identically with Kaula's[6] fourth-order expressions for $G_{\ell pq}(e)$ for $2 \leq \ell \leq 4$, $0 \leq p \leq 4$, and $|q| \leq 2$, except for the cases $(\ell, p, q) = (4, 1, -1)$ and $(4, 3, 1)$ where Kaula's second term should be $33e^3/16$ instead of $33e^2/16$. A comparison was also made with fourth-order results for $G_{2\ell p}(e)$ for $0 \leq p \leq 2$ and $|q| \leq 2$ given by Goldreich and Peale.[9] The agreement was identical except for the cases $(\ell, p, q) = (2, 0, -1)$ and $(2, 2, 1)$, where their first term should be $-e/2$ instead of $e/2$, and $(2, 2, -1)$ where their second term should be $-123e^3/16$ instead of $-123e^3/15$.

Consistency checks were also performed upon the MACSYMA formulae using the following relations:[10]

$$K_{\ell p,+1} = \frac{1}{2}(3\ell - 4p + 1) + O(e^2) \tag{7}$$

and

$$K_{\ell p,-1} = \frac{1}{2}(4p - \ell + 1) + O(e^2) \tag{8}$$

General overall agreement was attained to this order of accuracy.

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APPENDIX A

TABLE OF $K_{\ell pq}(e)$ ECCENTRICITY FUNCTION* FORMULAE FOR $2 \leqslant \ell \leqslant 10$, $0 \leqslant p \leqslant 5$, AND $|q| \leqslant 2$

*For the sake of brevity use is made of the property $K_{\ell pq} = K_{\ell, \ell-p, -q}$ in presenting the following tables.

$l_p q l_p q$	$K_{l_p q}$
2 0 -2 2 2 2	0
2 0 -1 2 2 1	$\frac{1}{2} - \frac{e^2}{16} + \frac{5e^4}{384} - \frac{143e^6}{18432} + \frac{9097e^8}{1474560} - \frac{878959e^{10}}{176947200} + \ldots$
2 0 0 2 2 0	$1 - \frac{5e^2}{2} + \frac{13e^4}{16} - \frac{35e^6}{288} + \frac{5e^8}{576} - \frac{49e^{10}}{3600} + \ldots$
2 0 1 2 2 -1	$\frac{7}{2} - \frac{123e^2}{16} + \frac{489e^4}{128} - \frac{1763e^6}{2048} + \frac{13527e^8}{163840} - \frac{180369e^{10}}{6553600} + \ldots$
2 0 2 2 2 -2	$\frac{17}{2} - \frac{115e^2}{6} + \frac{601e^4}{48} - \frac{1423e^6}{360} + \frac{48619e^8}{69120} - \frac{275587e^{10}}{2419200} + \ldots$
2 1 -2 2 1 2	$\frac{9}{4} - \frac{7e^2}{4} + \frac{141e^4}{64} - \frac{197e^6}{80} + \frac{62401e^8}{23040} - \frac{262841e^{10}}{89600} + \ldots$
2 1 -1 2 1 1	$\frac{3}{2} - \frac{27e^2}{16} + \frac{261e^4}{128} - \frac{14309e^6}{6144} + \frac{423907e^8}{163840} - \frac{55489483e^{10}}{19660800} + \ldots$
2 1 0 2 1 0	$-\frac{1}{\sqrt{(1-e^2)}\,(-1+e^2)}$
3 0 -2 3 3 2	$1 - \frac{e^2}{8} + \frac{55e^4}{48} - \frac{1177e^6}{3072} + \frac{172981e^8}{92160} - \frac{9657097e^{10}}{17694720} + \ldots$
3 0 -1 3 3 1	$-1 + \frac{5e^2}{4} - \frac{7e^4}{48} + \frac{23e^6}{288} - \frac{61e^8}{1440} + \frac{23453e^{10}}{691200} + \ldots$

$l_{pq}l_{pq}$	K_{lpq}
3 0 0 3 3 0	$1 - 6e^2 + \frac{423}{64}e^4 - \frac{125}{64}e^6 + \frac{7533}{16384}e^8 - \frac{81}{1600}e^{10} + \ldots$
3 0 1 3 3 -1	$5 - 22e^2 + \frac{607}{24}e^4 - \frac{98}{9}e^6 + \frac{32753}{11520}e^8 - \frac{4669}{17280}e^{10} + \ldots$
3 0 2 3 3 -2	$\frac{127}{8} - \frac{3065}{48}e^2 + \frac{243805}{3072}e^4 - \frac{798865}{18432}e^6 + \frac{49537175}{3538944}e^8 - \frac{130509569}{49545216}e^{10} + \ldots$
3 1 -2 3 2 2	$\frac{11}{8} + \frac{49}{16}e^2 + \frac{15665}{3072}e^4 + \frac{684503}{92160}e^6 + \frac{3937739}{393216}e^8 + \frac{15899369747}{1238630400}e^{10} + \ldots$
3 1 -1 3 2 1	$\frac{1}{\sqrt{(1-e^2)}\,(1-2e^2+e^4)}$
3 1 0 3 2 0	$1 + 2e^2 + \frac{239}{64}e^4 + \frac{3323}{576}e^6 + \frac{1193893}{147456}e^8 + \frac{4922717}{460800}e^{10} + \ldots$
3 1 1 3 2 -1	$3 + \frac{11}{4}e^2 + \frac{245}{48}e^4 + \frac{463}{64}e^6 + \frac{27929}{2880}e^8 + \frac{4286563}{345600}e^{10} + \ldots$
3 1 2 3 2 -2	$\frac{53}{8} + \frac{39}{16}e^2 + \frac{7041}{1024}e^4 + \frac{89317}{10240}e^6 + \frac{7447867}{655360}e^8 + \frac{650531047}{45875200}e^{10} + \ldots$
4 0 -2 4 4 2	$\frac{1}{2} - \frac{3}{8}e^2 - \frac{11}{360}e^6 + \frac{1477}{69120}e^8 - \frac{6989}{403200}e^{10} + \ldots$

I_{pqlpq}	K_{lpq}
4 0 -1 4 4 1	$-\frac{3}{2} + \frac{75\,e^2}{16} - \frac{393\,e^4}{128} + \frac{1251\,e^6}{2048} - \frac{25371\,e^8}{163840} + \frac{275439\,e^{10}}{6553600} + \ldots$
4 0 0 4 4 0	$1 - 11\,e^2 + \frac{199\,e^4}{8} - \frac{655\,e^6}{36} + \frac{13325\,e^8}{2304} - \frac{72461\,e^{10}}{57600} + \ldots$
4 0 1 4 4 -1	$-\frac{13}{2} + \frac{765\,e^2}{16} - \frac{37925\,e^4}{384} + \frac{1493005\,e^6}{18432} - \frac{3289075\,e^8}{98304} + \frac{62149637\,e^{10}}{7077888} + \ldots$
4 0 2 4 4 -2	$-\frac{51}{2} + \frac{321\,e^2}{2} - \frac{2613\,e^4}{8} + \frac{47583\,e^6}{160} - \frac{379419\,e^8}{2560} + \frac{4247043\,e^{10}}{89600} + \ldots$
4 1 -2 4 3 2	$-\frac{3}{\sqrt{1-e^2}\,(-4 + 12\,e^2 - 12\,e^4 + 4\,e^6)}$
4 1 -1 4 3 1	$-1 + \frac{33\,e^2}{16} + \frac{1865\,e^4}{384} + \frac{169229\,e^6}{18432} + \frac{499765\,e^8}{32768} + \frac{4116160969\,e^{10}}{176947200} - \ldots$
4 1 0 4 3 0	$1 + e^2 + \frac{65\,e^4}{16} + \frac{139\,e^6}{18} + \frac{30311\,e^8}{2304} + \frac{588173\,e^{10}}{28800} + \ldots$
4 1 1 4 3 -1	$-\frac{9}{2} + \frac{3\,e^2}{16} + \frac{963\,e^4}{128} + \frac{22737\,e^6}{2048} + \frac{2927457\,e^8}{163840} + \frac{172753887\,e^{10}}{6553600} + \ldots$
4 1 2 4 3 -2	$-\frac{53}{4} + \frac{179\,e^2}{24} + \frac{977\,e^4}{64} + \frac{81553\,e^6}{5760} + \frac{649547\,e^8}{27648} + \frac{35552851\,e^{10}}{1075200} + \ldots$
4 2 -2 4 2 2	$5 + \frac{155\,e^2}{12} + \frac{835\,e^4}{32} + \frac{12953\,e^6}{288} + \frac{242797\,e^8}{3456} + \frac{5511593\,e^{10}}{53760} + \ldots$

$\|p q\|p q$	$K_{\|p q}$
9 3 2 9 6 -2	$\frac{301}{8} + \frac{3743\,e^2}{24} + \frac{2016079\,e^4}{3072} + \frac{4823461\,e^6}{2304} + \frac{20321424821\,e^8}{3538944} + \frac{346318732157\,e^{10}}{24772608} + \ldots$
9 4 -2 9 5 2	$\frac{89}{8} + \frac{961\,e^2}{8} + \frac{2072387\,e^4}{3072} + \frac{30799309\,e^6}{11520} + \frac{49909418551\,e^8}{5898240} + \frac{14144477757409\,e^{10}}{619315200} + \ldots$
9 4 -1 9 5 1	$(64 + 336\,e^2 + 280\,e^4 + 35\,e^6) / (\mathrm{sqrt}(1 - e^2)\,(16 - 128\,e^2 + 448\,e^4 - 896\,e^6 + 1120\,e^8 - 896\,e^{10} + 448\,e^{12} - 128\,e^{14} + 16\,e^{16}))$
9 4 0 9 5 0	$1 + \frac{43\,e^2}{2} + \frac{11087\,e^4}{64}/ + \frac{994855\,e^6}{1152} + \frac{473226805\,e^8}{147456} + \frac{71957389547\,e^{10}}{7372800} + \ldots$
9 4 1 9 5 -1	$6 + \frac{145\,e^2}{2} + \frac{10531\,e^4}{24} + \frac{88091\,e^6}{48} + \frac{34856761\,e^8}{5760} + \frac{728862887\,e^{10}}{43200} + \ldots$
9 4 2 9 5 -2	$\frac{179}{8} + \frac{1559\,e^2}{8} + \frac{995215\,e^4}{1024} + \frac{2284221\,e^6}{640} + \frac{7022778909\,e^8}{655360} + \frac{638446109013\,e^{10}}{22937600} + \ldots$

$l_{pq}l_{pq}$	K_{lpq}
9 2 -1 9 7 1	$5e + \frac{2}{3} + \frac{4}{3}\frac{65e}{} + \frac{6}{}\frac{265e}{} + \frac{8}{288}\frac{80915e}{} + \frac{10}{6912}\frac{5296655e}{} + \ldots$
9 2 0 9 7 0	$1 - \frac{2}{2}\frac{5e}{} + \frac{4}{64}\frac{1655e}{} + \frac{6}{1152}\frac{69055e}{} + \frac{8}{147456}\frac{32227525e}{} + \frac{10}{294912}\frac{177233315e}{} + \ldots$
9 2 1 9 7 -1	$10 - \frac{2}{2}\frac{55e}{} + \frac{4}{8}\frac{915e}{} + \frac{6}{32}\frac{3775e}{} + \frac{8}{128}\frac{62827e}{} + \frac{10}{2560}\frac{3005819e}{} + \ldots$
9 2 2 9 7 -2	$\frac{455}{8} - \frac{2}{8}\frac{1365e}{} + \frac{4}{3072}\frac{1432025e}{} + \frac{6}{1152}\frac{103243e}{} + \frac{8}{1179648}\frac{1264544029e}{} + \frac{10}{17694720}\frac{37457929817e}{} + \ldots$
9 3 -2 9 6 2	$\frac{31}{8} + \frac{2}{24}\frac{1001e}{} + \frac{4}{3072}\frac{720481e}{} + \frac{6}{5760}\frac{5369863e}{} + \frac{8}{3538944}\frac{10479640175e}{} + \frac{10}{619315200}\frac{4968366161891e}{} + \ldots$
9 3 -1 9 6 1	$2 + \frac{2}{2}\frac{49e}{} + \frac{4}{24}\frac{3589e}{} + \frac{6}{288}\frac{181457e}{} + \frac{8}{5760}\frac{12045013e}{} + \frac{10}{345600}\frac{2026653877e}{} + \ldots$
9 3 0 9 6 0	$1 + \frac{2}{2}\frac{27e}{} + \frac{4}{64}\frac{5895e}{} + \frac{6}{128}\frac{53239e}{} + \frac{8}{16384}\frac{23783421e}{} + \frac{10}{819200}\frac{3465568467e}{} + \ldots$
9 3 1 9 6 -1	$8 + 53e + \frac{2}{3} + \frac{4}{72}\frac{800e}{} + \frac{6}{9}\frac{71039e}{} + \frac{8}{172800}\frac{27043e}{} + \frac{10}{}\frac{1369364531e}{} + \ldots$

$l_{pq}l_{pq}$	K_{lpq}
9 0 2 9 9 -2	$\frac{859}{8} - \frac{64867}{24}e^{2} + \frac{73661797}{3072}e^{4} - \frac{605735603}{5760}e^{6} + \frac{4671361372183}{17694720}e^{8} - \frac{255917766969601}{619315200}e^{10} + \ldots$
9 1 -2 9 8 2	$\frac{11}{8} - \frac{49}{8}e^{2} + \frac{27509}{3072}e^{4} - \frac{1751}{1152}e^{6} + \frac{3831427}{393216}e^{8} - \frac{556580551}{24772608}e^{10} + \ldots$
9 1 -1 9 8 1	$-2 + \frac{41}{2}e^{2} - \frac{475}{8}e^{4} + \frac{591}{8}e^{6} - \frac{4509}{128}e^{8} + \frac{107079}{3200}e^{10} + \ldots$
9 1 0 9 8 0	$1 - \frac{53}{2}e^{2} + \frac{10655}{64}e^{4} - \frac{467369}{1152}e^{6} + \frac{71447509}{147456}e^{8} - \frac{2180918677}{7372800}e^{10} + \ldots$
9 1 1 9 8 -1	$12 - 193e^{2} + \frac{5851}{6}e^{4} - \frac{35141}{16}e^{6} + \frac{1897571}{720}e^{8} - \frac{78885121}{43200}e^{10} + \ldots$
9 1 2 9 8 -2	$\frac{641}{8} - \frac{8183}{8}e^{2} + \frac{4719017}{1024}e^{4} - \frac{12842891}{1280}e^{6} + \frac{1616611371}{131072}e^{8} - \frac{214509553077}{22937600}e^{10} + \ldots$
9 2 -2 9 7 2	$\frac{5}{8} + \frac{45}{8}e^{2} + \frac{31005}{1024}e^{4} + \frac{29885}{256}e^{6} + \frac{47673655}{131072}e^{8} + \frac{4464073067}{4587520}e^{10} + \ldots$

$l\,p\,q\,l\,p\,q$	K_{lpq}
8 4 -1 8 4 1	$\frac{9}{2} + \frac{783\,e^2}{16} + \frac{33795\,e^4}{128} + \frac{2025143\,e^6}{2048} + \frac{481921089\,e^8}{163840} + \frac{48852422813\,e^{10}}{6553600} + \ldots$
8 4 0 8 4 0	$-(16 + 168\,e^2 + 210\,e^4 + 35\,e^6)$ $/(\sqrt{1-e^2}\,(-16 + 112\,e^2 - 336\,e^4 + 560\,e^6 - 560\,e^8 + 336\,e^{10} - 112\,e^{12} + 16\,e^{14}))$
9 0 -2 9 9 2	$\frac{49}{8} - \frac{1813\,e^2}{24} + \frac{940555\,e^4}{3072} - \frac{6417187\,e^6}{11520} + \frac{9184555933\,e^8}{17694720} - \frac{24711168097\,e^{10}}{88473600} + \ldots$
9 0 -1 9 9 1	$-4 + 95\,e^2 - \frac{4121\,e^4}{6} + \frac{317263\,e^6}{144} - \frac{2638483\,e^8}{720} + \frac{149923637\,e^{10}}{43200} + \ldots$
9 0 0 9 9 0	$1 - \frac{117\,e^2}{2} + \frac{51327\,e^4}{64} - \frac{573825\,e^6}{128} + \frac{207360045\,e^8}{16384} - \frac{16531322157\,e^{10}}{819200} + \ldots$
9 0 1 9 9 -1	$14 - \frac{935\,e^2}{2} + \frac{117175\,e^4}{24} - \frac{846305\,e^6}{36} + \frac{70993175\,e^8}{1152} - \frac{333965303\,e^{10}}{3456} + \ldots$

$l_{pq}l_{pq}$	K_{lpq}
8 2 2 8 6 -2	$42 - \frac{149\,e^2}{4} + \frac{7005\,e^4}{32} + \frac{51513\,e^6}{160} + \frac{223569\,e^8}{256} + \frac{46990503\,e^{10}}{25600} + \ldots$
8 3 -2 8 5 2	$-(336 + 560\,e^2 + 105\,e^4)/(\mathrm{sqrt}(1 - e^2)(-64 + 448\,e^2 - 1344\,e^4 + 2240\,e^6 - 2240\,e^8 + 1344\,e^{10} - 448\,e^{12} + 64\,e^{14}))$
8 3 -1 8 5 1	$5 - \frac{433\,e^2}{16} + \frac{56077\,e^4}{384} + \frac{10099021\,e^6}{18432} + \frac{2408719349\,e^8}{1474560} + \frac{146849722309\,e^{10}}{35389440} + \ldots$
8 3 0 8 5 0	$1 + 14\,e^2 + \frac{1387\,e^4}{16} + \frac{51035\,e^6}{144} + \frac{1292339\,e^8}{1152} + \frac{171620111\,e^{10}}{57600} + \ldots$
8 3 1 8 5 -1	$\frac{13}{2} + \frac{765\,e^2}{16} + \frac{28143\,e^4}{128} + \frac{1534433\,e^6}{2048} + \frac{343550181\,e^8}{163840} + \frac{33392327199\,e^{10}}{6553600} + \ldots$
8 3 2 8 5 -2	$\frac{103}{4} + \frac{3053\,e^2}{24} + \frac{46325\,e^4}{96} + \frac{4144921\,e^6}{2880} + \frac{506333237\,e^8}{138240} + \frac{80185662059\,e^{10}}{9676800} + \ldots$
8 4 -2 8 4 2	$\frac{27}{2} + 111\,e^2 + \frac{8127\,e^4}{16} + \frac{272871\,e^6}{160} + \frac{3603907\,e^8}{768} + \frac{502522513\,e^{10}}{44800} + \ldots$

$l_{pq}l_{pq}$	$K_{l_{pq}}$
8 1 0 8 7 0	$1 - 18\,e^{2} + \frac{1251\,e^{4}}{16} - \frac{1957\,e^{6}}{16} + \frac{3261\,e^{8}}{32} - \frac{126249\,e^{10}}{6400} + \ldots$
8 1 1 8 7 -1	$\frac{21}{2} - \frac{1911\,e^{2}}{16} + \frac{55055\,e^{4}}{128} - \frac{4070857\,e^{6}}{6144} + \frac{275593591\,e^{8}}{491520} - \frac{13561818053\,e^{10}}{58982400} + \ldots$
8 1 2 8 7 -2	$\frac{249}{4} - \frac{4643\,e^{2}}{8} + \frac{122621\,e^{4}}{64} - \frac{5731619\,e^{6}}{1920} + \frac{30655519\,e^{8}}{11520} - \frac{1122269891\,e^{10}}{806400} + \ldots$
8 2 -2 8 6 2	$1 + \frac{103\,e^{2}}{12} + \frac{3899\,e^{4}}{96} + \frac{50363\,e^{6}}{360} + \frac{848807\,e^{8}}{2160} + \frac{4623839569\,e^{10}}{4838400} + \ldots$
8 2 -1 8 6 1	$1 - \frac{99\,e^{2}}{16} + \frac{3795\,e^{4}}{128} + \frac{216251\,e^{6}}{2048} + \frac{49870857\,e^{8}}{163840} + \frac{4956626553\,e^{10}}{6553600} + \ldots$
8 2 0 8 6 0	$1 + 2\,e^{2} + \frac{47\,e^{4}}{2} + \frac{707\,e^{6}}{9} + \frac{540877\,e^{8}}{2304} + \frac{8581691\,e^{10}}{14400} + \ldots$
8 2 1 8 6 -1	$\frac{17}{2} - \frac{5\,e^{2}}{16} / \frac{28345\,e^{4}}{384} + \frac{3196075\,e^{6}}{18432} + \frac{138380365\,e^{8}}{294912} + \frac{7635333395\,e^{10}}{7077888} + \frac{3579441729\,e^{10}}{179200} + \ldots$

$l_{pq}l_{pq}$	$K_{l_{pq}}$
8 0 -2 8 8 2	$\frac{9}{2} - \frac{81}{2}e^{2} + \frac{1845}{16}e^{4} - \frac{11169}{80}e^{6} + \frac{106389}{1280}e^{8} - \frac{1345311}{44800}e^{10} + \ldots$
8 0 -1 8 7 1	$-\frac{7}{2} + \frac{1015}{16}e^{2} - \frac{131831}{384}e^{4} + \frac{14758807}{18432}e^{6} - \frac{1379540071}{1474560}e^{8} + \frac{100056815223}{176947200}e^{10} + \ldots$
8 0 0 8 8 0	$1 - 46\,e^{2} + \frac{1963}{4}e^{4} - \frac{75595}{36}e^{6} + \frac{5113919}{1152}e^{8} - \frac{37315013}{7200}e^{10} + \ldots$
8 0 1 8 8 -1	$-\frac{25}{2} + \frac{5337}{16}e^{2} - \frac{352899}{128}e^{4} + \frac{21222321}{2048}e^{6} - \frac{3416816241}{163840}e^{8} + \frac{6483396851}{262144}e^{10} + \ldots$
8 0 2 8 8 -2	$-\frac{173}{2} + \frac{10585}{6}e^{2} - \frac{603415}{48}e^{4} + \frac{3152105}{72}e^{6} - \frac{595640575}{6912}e^{8} + \frac{5065352177}{48384}e^{10} + \ldots$
8 1 -2 8 7 2	$\frac{3}{4} - \frac{13}{8}e^{2} + \frac{5}{2}e^{4} - \frac{491}{120}e^{6} + \frac{115411}{9216}e^{8} - \frac{98179673}{3225600}e^{10} + \ldots$
8 1 -1 8 7 1	$-\frac{3}{2} + \frac{165}{16}e^{2} - \frac{2225}{128}e^{4} + \frac{99355}{6144}e^{6} - \frac{542995}{98304}e^{8} + \frac{61204775}{2359296}e^{10} + \ldots$

I_{pqlpq}	K_{lpq}
7 2 1 7 5 -1	$7 + \frac{27e^2}{2} + \frac{385e^4}{6} + \frac{24509e^6}{144} + \frac{1548977e^8}{3840} + \frac{294026953e^{10}}{345600} + \ldots$
7 2 2 7 5 -2	$\frac{235}{8} + \frac{1025e^2}{48} + \frac{147075e^4}{1024} + \frac{5723405e^6}{18432} + \frac{2410472275e^8}{3538944} + \frac{22130823275e^{10}}{16515072} + \ldots$
7 3 -2 7 4 2	$\frac{55}{8} + \frac{2405e^2}{48} + \frac{207871e^4}{1024} + \frac{56046421e^6}{92160} + \frac{26655858811e^8}{17694720} + \frac{269963728819e^{10}}{82575360} + \ldots$
7 3 -1 7 4 1	$\frac{24 + 60e^2 + 15e^4}{\sqrt{(1-e^2)}\,(8 - 48e^2 + 120e^4 - 160e^6 + 120e^8 - 48e^{10} + 8e^{12})}$
7 3 0 7 4 0	$1 + 13e^2 + \frac{4535e^4}{64} + \frac{18397e^6}{72} + \frac{106083029e^8}{147456} + \frac{6324397591e^{10}}{3686400} + \ldots$
7 3 1 7 4 -1	$5 + \frac{147e^2}{4} + \frac{7369e^4}{48} + \frac{136175e^6}{288} + \frac{4604159e^8}{3840} + \frac{45857329e^{10}}{17280} + \ldots$
7 3 2 7 4 -2	$\frac{129}{8} + \frac{1353e^2}{16} + \frac{302853e^4}{1024} + \frac{8305863e^6}{10240} + \frac{1242373407e^8}{655360} + \frac{181184734241e^{10}}{45875200} + \ldots$

$l_{pq} l_{pq}$	$K_{l_{pq}}$
7 1 -2 7 6 2	$\frac{3}{8} + \frac{3\,e^2}{16} + \frac{2331\,e^4}{1024} + \frac{68481\,e^6}{10240} + \frac{11060469\,e^8}{655360} + \frac{1706068467\,e^{10}}{45875200} + \ldots$
7 1 -1 7 6 1	$-1 + \frac{9\,e^2}{2} - \frac{13\,e^4}{6} + \frac{241\,e^6}{36} + \frac{51577\,e^8}{3840} + \frac{10650431\,e^{10}}{345600} + \ldots$
7 1 0 7 6 0	$1 - 11\,e^2 + \frac{2015\,e^4}{64} - \frac{220\,e^6}{9} + \frac{3523685\,e^8}{147456} + \frac{3230695\,e^{10}}{147456} + \ldots$
7 1 1 7 6 -1	$9 - \frac{267\,e^2}{4} + \frac{2607\,e^4}{16} - \frac{9651\,e^6}{64} + \frac{134619\,e^8}{1280} + \frac{399951\,e^{10}}{25600} + \ldots$
7 1 2 7 6 -2	$\frac{373}{8} - \frac{14281\,e^2}{48} + \frac{703881\,e^4}{1024} - \frac{65327381\,e^6}{92160} + \frac{8398039433\,e^8}{17694720} - \frac{5944612901\,e^{10}}{58982400} + \ldots$
7 2 -2 7 5 2	$\frac{13}{8} + \frac{575\,e^2}{48} + \frac{50273\,e^4}{1024} + \frac{13696627\,e^6}{92160} + \frac{6574316609\,e^8}{17694720} + \frac{37289822893\,e^{10}}{45875200} + \ldots$
7 2 -1 7 5 1	$1 + \frac{33\,e^2}{4} + \frac{1709\,e^4}{48} + \frac{64559\,e^6}{576} + \frac{1109107\,e^8}{3840} + \frac{447496337\,e^{10}}{691200} + \ldots$
7 2 0 7 5 0	$1 + 5\,e^2 + \frac{1647\,e^4}{64} + \frac{335\,e^6}{4} + \frac{3652765\,e^8}{16384} + \frac{209965191\,e^{10}}{409600} + \ldots$

$l_{pq}l_{pq}$	$K_{l_{pq}}$
6 3 -1 6 3 1	$-\frac{7}{2} + \frac{371\,e^2}{16} + \frac{32515\,e^4}{384} + \frac{4225655\,e^6}{18432} + \frac{761278399\,e^8}{1474560} + \frac{181404893243\,e^{10}}{176947200} + \ldots$
6 3 0 6 3 0	$-\frac{8 + 40\,e^2 + 15\,e^4}{\sqrt{(1-e^2)}\,(-8 + 40\,e^2 - 80\,e^4 + 80\,e^6 - 40\,e^8 + 8\,e^{10})}$
7 0 -2 7 7 2	$\frac{25}{8} - \frac{925\,e^2}{48} + \frac{36125\,e^4}{1024} - \frac{463525\,e^6}{18432} + \frac{31680025\,e^8}{3538944} - \frac{11921725\,e^{10}}{5505024} + \ldots$
7 0 -1 7 7 1	$-3 + \frac{159\,e^2}{4} - \frac{2445\,e^4}{16} + \frac{969\,e^6}{4} - \frac{235401\,e^8}{1280} + \frac{505257\,e^{10}}{6400} + \ldots$
7 0 0 7 7 0	$1 - 35\,e^2 + \frac{17927\,e^4}{64} - \frac{63245\,e^6}{72} + \frac{194651765\,e^8}{147456} - \frac{3940556921\,e^{10}}{3686400} + \ldots$
7 0 1 7 7 -1	$11 - 228\,e^2 + \frac{17341\,e^4}{12} - \frac{36794\,e^6}{9} + \frac{3880157\,e^8}{640} - \frac{56415557\,e^{10}}{10800} + \ldots$
7 0 2 7 7 -2	$\frac{543}{8} - \frac{17505\,e^2}{16} + \frac{6238503\,e^4}{1024} - \frac{167296347\,e^6}{10240} + \frac{15988224537\,e^8}{655360} - \frac{1022767782417\,e^{10}}{45875200} + \ldots$

$l_{pq}l_{pq}$	$K_{l_{pq}}$
6 1 1 6 5 -1	$\frac{15}{2} - \frac{505\,e^2}{16} + \frac{19795\,e^4}{384} - \frac{32125\,e^6}{2048} + \frac{9717515\,e^8}{294912} - \frac{303515015\,e^{10}}{7077888} + \ldots$
6 1 2 6 5 -2	$\frac{133}{4} - 131\,e^2 + \frac{13105\,e^4}{64} - \frac{36219\,e^6}{320} + \frac{93879\,e^8}{1024} - \frac{676209\,e^{10}}{12800} + \ldots$
6 2 -2 6 4 2	$-\frac{10 + 5\,e^2}{\sqrt{1-e^2}\,(-4 + 20\,e^2 - 40\,e^4 + 40\,e^6 - 20\,e^8 + 4\,e^{10})}$
6 2 -1 6 4 1	$\frac{3}{2} + \frac{161\,e^2}{16} + \frac{14239\,e^4}{384} + \frac{621595\,e^6}{6144} + \frac{67628063\,e^8}{294912} + \frac{81019919057\,e^{10}}{176947200} + \ldots$
6 2 0 6 4 0	$1 + \frac{13\,e^2}{2} + \frac{419\,e^4}{16} + \frac{21697\,e^6}{288} + \frac{408937\,e^8}{2304} + \frac{42045317\,e^{10}}{115200} + \ldots$
6 2 1 6 4 -1	$\frac{11}{2} + \frac{277\,e^2}{16} + \frac{7093\,e^4}{128} + \frac{278093\,e^6}{2048} + \frac{47240923\,e^8}{163840} + \frac{3609377983\,e^{10}}{6553600} + \ldots$
6 2 2 6 4 -2	$19 + 35\,e^2 + \frac{4997\,e^4}{48} + \frac{40543\,e^6}{180} + \frac{5098781\,e^8}{11520} + \frac{964570001\,e^{10}}{1209600} + \ldots$
6 3 -2 6 3 2	$\frac{35}{4} + \frac{133\,e^2}{3} + \frac{6727\,e^4}{48} + \frac{498547\,e^6}{1440} + \frac{10115105\,e^8}{13824} + \frac{47930237\,e^{10}}{34560} + \ldots$

$l_{pq}l_{pq}$	$K_{l_{pq}}$
6 0 -2 6 6 2	$2 - \frac{23}{3}e^2 + \frac{185}{24}e^4 - \frac{1909}{720}e^6 + \frac{2399}{3456}e^8 + \frac{13423}{1209600}e^{10} + \ldots$
6 0 -1 6 6 1	$-\frac{5}{2} + \frac{365}{16}e^2 - \frac{22145}{384}e^4 + \frac{1030925}{18432}e^6 - \frac{7379545}{294912}e^8 + \frac{50465285}{7077888}e^{10} + \ldots$
6 0 0 6 6 0	$1 - \frac{51}{2}e^2 + \frac{2331}{16}e^4 - \frac{10079}{32}e^6 + \frac{80043}{256}e^8 - \frac{2114073}{12800}e^{10} + \ldots$
6 0 1 6 6 -1	$-\frac{19}{2} + \frac{2359}{16}e^2 - \frac{263935}{384}e^4 + \frac{25687879}{18432}e^6 - \frac{2128113547}{1474560}e^8 + \frac{152987554951}{176947200}e^{10} + \ldots$
6 0 2 6 6 -2	$-\frac{103}{2} + \frac{1907}{3}e^2 - \frac{64079}{24}e^4 + \frac{1907333}{360}e^6 - \frac{199062983}{34560}e^8 + \frac{1157658389}{302400}e^{10} + \ldots$
6 1 -2 6 5 2	$1 + e^2 + \frac{677}{192}e^4 - \frac{26671}{2880}e^6 + \frac{104891}{5120}e^8 - \frac{97317793}{2419200}e^{10} + \ldots$
6 1 -1 6 5 1	$-\frac{1}{2} + \frac{31}{16}e^2 - \frac{305}{128}e^4 + \frac{15579}{2048}e^6 - \frac{2779023}{163840}e^8 + \frac{221939397}{6553600}e^{10} + \ldots$
6 1 0 6 5 0	$1 - \frac{11}{2}e^2 + 11e^4 - \frac{17}{9}e^6 + \frac{34897}{2304}e^8 - \frac{3260153}{115200}e^{10} + \ldots$

$l_{pq}l_{pq}$	K_{lpq}
5 1 0 5 4 0	$1 - \frac{3e^2}{2} + \frac{303e^4}{64} + \frac{925e^6}{128} + \frac{248493e^8}{16384} + \frac{22151757e^{10}}{819200} + \ldots$
5 1 1 5 4 -1	$6 - \frac{21e^2}{2} + \frac{31e^4}{2} + \frac{929e^6}{96} + \frac{45443e^8}{1920} + \frac{4444903e^{10}}{115200} + \ldots$
5 1 2 5 4 -2	$\frac{177}{8} - \frac{177e^2}{4} + \frac{52945e^4}{1024} + \frac{7739e^6}{3072} + \frac{14847329e^8}{393216} + \frac{108103973e^{10}}{2064384} + \ldots$
5 2 -2 5 3 2	$\frac{29}{8} + \frac{193e^2}{12} + \frac{135175e^4}{3072} + \frac{4381873e^6}{46080} + \frac{3150020521e^8}{17694720} + \frac{46806205751e^{10}}{154828800} + \ldots$
5 2 -1 5 3 1	$\frac{4 + 3e^2}{\sqrt{(1-e^2)}\,(2 - 8e^2 + 12e^4 - 8e^6 + 2e^8)}$
5 2 0 5 3 0	$1 + \frac{13e^2}{2} + \frac{1399e^4}{64} + \frac{61837e^6}{1152} + \frac{16171237e^8}{147456} + \frac{1463469397e^{10}}{7372800} + \ldots$
5 2 1 5 3 -1	$4 + \frac{29e^2}{2} + \frac{467e^4}{12} + \frac{24181e^6}{288} + \frac{909551e^8}{5760} + \frac{46630727e^{10}}{172800} + \ldots$
5 2 2 5 3 -2	$\frac{87}{8} + \frac{107e^2}{4} + \frac{64539e^4}{1024} + \frac{633603e^6}{5120} + \frac{143148297e^8}{655360} + \frac{127755531e^{10}}{358400} + \ldots$

$\lvert p q \rvert p q$	$K_{\lvert p q}$
4 2 -1 4 2 1	$-\frac{5}{2} + \frac{135\,e^2}{16} + \frac{7285\,e^4}{384} + \frac{643015\,e^6}{18432} + \frac{5591339\,e^8}{98304} + \frac{3028339579\,e^{10}}{35389440} + \ldots$
4 2 0 4 2 0	$-\frac{2 + 3\,e^2}{\sqrt{1 - e^2}\,(-2 + 6\,e^2 - 6\,e^4 + 2\,e^6)}$
5 0 -2 5 5 2	$-\frac{9}{8} - \frac{9\,e^2}{4} + \frac{873\,e^4}{1024} - \frac{891\,e^6}{5120} + \frac{3807\,e^8}{655360} - \frac{7029\,e^{10}}{819200} + \ldots$
5 0 -1 5 5 1	$-2 + \frac{23\,e^2}{2} - \frac{101\,e^4}{6} + \frac{2401\,e^6}{288} - \frac{12799\,e^8}{5760} + \frac{104137\,e^{10}}{345600} + \ldots$
5 0 0 5 5 0	$1 - \frac{35\,e^2}{2} + \frac{4255\,e^4}{64} - \frac{104075\,e^6}{1152} + \frac{7873525\,e^8}{147456} - \frac{5269187\,e^{10}}{294912} + \ldots$
5 0 1 5 5 -1	$8 - \frac{177\,e^2}{2} + \frac{573\,e^4}{2} - \frac{3109\,e^6}{8} + \frac{168009\,e^8}{640} - \frac{1352919\,e^{10}}{12800} + \ldots$
5 0 2 5 5 -2	$-\frac{299}{8} - \frac{4067\,e^2}{12} + \frac{3146269\,e^4}{3072} - \frac{65724449\,e^6}{46080} + \frac{19022800279\,e^8}{17694720} - \frac{1393243901\,e^{10}}{2764800} + \ldots$
5 1 -2 5 4 2	$-\frac{3}{8} + \frac{7\,e^2}{4} + \frac{5111\,e^4}{1024} + \frac{170717\,e^6}{15360} + \frac{125537797\,e^8}{5898240} + \frac{118680949\,e^{10}}{3225600} + \ldots$
5 1 -1 5 4 1	$\frac{3\,e^2}{2} + 4\,e^4 + \frac{147\,e^6}{16} + \frac{6899\,e^8}{384} + \frac{727813\,e^{10}}{23040} + \ldots$

$l_{pq}l_{pq}$	K_{lpq}
10 0 -2 10 10 2	$8 - \frac{388\,e^2}{3} + \frac{1409\,e^4}{2} - \frac{321229\,e^6}{180} + \frac{2594831\,e^8}{1080} - \frac{47407249\,e^{10}}{25200} + \ldots$
10 0 -1 10 10 1	$\frac{9}{2} - \frac{2169\,e^2}{16} + \frac{161343\,e^4}{128} - \frac{10868481\,e^6}{2048} + \frac{1946663721\,e^8}{163840} - \frac{101493784107\,e^{10}}{6553600} + \ldots$
10 0 0 10 10 0	$1 - \frac{145\,e^2}{2} + \frac{19865\,e^4}{16} - \frac{2527475\,e^6}{288} + \frac{36684775\,e^8}{1152} - \frac{153124733\,e^{10}}{2304} + \ldots$
10 0 1 10 10 -1	$\frac{31}{2} - \frac{10131\,e^2}{16} + \frac{3129731\,e^4}{384} - \frac{901040371\,e^6}{18432} + \frac{79489366957\,e^8}{491520} - \frac{57385925962211\,e^{10}}{176947200} + \ldots$
10 0 2 10 10 -2	$\frac{261}{2} - \frac{7947\,e^2}{2} + \frac{686997\,e^4}{16} - \frac{1157274\,e^6}{5} + \frac{1847400507\,e^8}{2560} - \frac{126939453777\,e^{10}}{89600} + \ldots$
10 1 -2 10 9 2	$\frac{9}{4} - \frac{63\,e^2}{4} + \frac{2241\,e^4}{64} - \frac{4833\,e^6}{160} + \frac{12987\,e^8}{640} - \frac{495207\,e^{10}}{44800} + \ldots$
10 1 -1 10 9 1	$\frac{5}{2} - \frac{579\,e^2}{16} + \frac{59689\,e^4}{384} - \frac{5312279\,e^6}{18432} + \frac{25620931\,e^8}{98304} - \frac{5441780687\,e^{10}}{35389440} + \ldots$

$l_{pq}l_{pq}$	$K_{l_{pq}}$
10 1 0 10 9 0	$1 - \frac{73\,e^2}{2} + \frac{2521\,e^4}{8} - \frac{156895\,e^6}{144} + \frac{67187\,e^8}{36} - \frac{1582691\,e^{10}}{900} + \ldots$
10 1 1 10 9 -1	$\frac{27}{2} - \frac{4653\,e^2}{16} + \frac{252189\,e^4}{128} - \frac{12448593\,e^6}{2048} + \frac{1652177691\,e^8}{163840} - \frac{64905473151\,e^{10}}{6553600} + \ldots$
10 1 2 10 9 -2	$\frac{401}{4} - \frac{20101\,e^2}{12} + \frac{633685\,e^4}{64} - \frac{8264365\,e^6}{288} + \frac{163340485\,e^8}{3456} - \frac{780875869\,e^{10}}{16128} + \ldots$
10 2 -2 10 8 2	$\frac{1}{2} + \frac{19\,e^2}{6} + \frac{335\,e^4}{16} + \frac{3992\,e^6}{45} + \frac{20986301\,e^8}{69120} + \frac{238193761\,e^{10}}{268800} + \ldots$
10 2 -1 10 8 1	$-\frac{1}{2} + \frac{93\,e^2}{16} + \frac{4291\,e^4}{384} + \frac{1274365\,e^6}{18432} + \frac{7647523\,e^8}{32768} + \frac{491941387\,e^{10}}{7077888} + \ldots$
10 2 0 10 8 0	$1 - \frac{17\,e^2}{2} + \frac{681\,e^4}{16} + \frac{437\,e^6}{32} + \frac{25741\,e^8}{128} + \frac{3430701\,e^{10}}{6400} + \ldots$
10 2 1 10 8 -1	$\frac{23}{2} - \frac{1143\,e^2}{16} + \frac{94339\,e^4}{384} - \frac{1984871\,e^6}{18432} + \frac{286202917\,e^8}{491520} - \frac{19166170127\,3\,e^{10}}{176947200} + \ldots$

$l_{pq}l_{pq}$	$K_{l_{pq}}$
10 2 2 10 8 -2	$74 - \frac{1246\,e^{2}}{3} + \frac{9563\,e^{4}}{8} - \frac{179773\,e^{6}}{180} + \frac{1998401\,e^{8}}{1080} + \frac{45645053\,e^{10}}{25200} + \ldots$
10 3 -2 10 7 2	$\frac{11}{4} + \frac{401\,e^{2}}{12} + \frac{6747\,e^{4}}{32} + \frac{1343161\,e^{6}}{1440} + \frac{452148163\,e^{8}}{138240} + \frac{15659474077\,e^{10}}{1612800} + \ldots$
10 3 -1 10 7 1	$\frac{3}{2} + \frac{327\,e^{2}}{16} + \frac{17613\,e^{4}}{128} + \frac{1308531\,e^{6}}{2048} + \frac{381139059\,e^{8}}{163840} + \frac{46648204029\,e^{10}}{6553600} + \ldots$
10 3 0 10 7 0	$1 + \frac{23\,e^{2}}{2} + \frac{703\,e^{4}}{8} + \frac{61919\,e^{6}}{144} + \frac{3762383\,e^{8}}{2304} + \frac{594945653\,e^{10}}{115200} + \ldots$
10 3 1 10 7 -1	$\frac{19}{2} + \frac{783\,e^{2}}{16} + \frac{109991\,e^{4}}{384} + \frac{20778515\,e^{6}}{18432} + \frac{364006933\,e^{8}}{98304} + \frac{74380337167\,e^{10}}{7077888} + \ldots$
10 3 2 10 7 -2	$\frac{207}{4} + \frac{591\,e^{2}}{4} + \frac{6315\,e^{4}}{8} + \frac{417141\,e^{6}}{160} + \frac{39309261\,e^{8}}{5120} + \frac{13098171383\,e^{10}}{201600} + \ldots$

$l_{pq}l_{pq}$	K_{lpq}
10 4 -2 10 6 2	$-(288 + 1008\,e^2 + 630\,e^4 + 63\,e^6)$ $/(\sqrt{1-e^2}\,(-32 + 288\,e^2 - 1152\,e^4 + 2688\,e^6 - 4032\,e^8 + 4032\,e^{10}$ $- 2688\,e^{12} + 1152\,e^{14} - 288\,e^{16} + 32\,e^{18}))$
10 4 -1 10 6 1	$\frac{7}{2} + \frac{897\,e^2}{16} + \frac{160547\,e^4}{384} + \frac{38138513\,e^6}{18432} + \frac{1288138311\,e^8}{163840} + \frac{4385825119633\,e^{10}}{176947200} + \ldots$
10 4 0 10 6 0	$1 + \frac{47\,e^2}{2} + \frac{3377\,e^4}{16} + \frac{337285\,e^6}{288} + \frac{1386901\,e^8}{288} + \frac{463529633\,e^{10}}{28800} + \ldots$
10 4 1 10 6 -1	$\frac{15}{2} + \frac{1509\,e^2}{16} + \frac{79137\,e^4}{128} + \frac{5772381\,e^6}{2048} + \frac{331616595\,e^8}{32768} + \frac{40132348803\,e^{10}}{1310720} + \ldots$
10 4 2 10 6 -2	$\frac{67}{2} + \frac{1753\,e^2}{6} + \frac{6191\,e^4}{4} + \frac{274574\,e^6}{45} + \frac{1362223271\,e^8}{69120} + \frac{14785480307\,e^{10}}{268800} + \ldots$
10 5 -2 10 5 2	$\frac{77}{4} + \frac{2783\,e^2}{12} + \frac{92763\,e^4}{64} + \frac{9146203\,e^6}{1440} + \frac{762793537\,e^8}{34560} + \frac{13098171383\,e^{10}}{201600} + \ldots$

$l_{pq}l_{pq}$	K_{lpq}
10 5 -1 10 5 1	$\frac{11}{2} + \frac{1419\,e^2}{16} + \frac{254551\,e^4}{384} + \frac{60493279\,e^6}{18432} + \frac{6126632897\,e^8}{491520} + \frac{6947140035539\,e^{10}}{176947200} + \ldots$
10 5 0 10 5 0	$-(128 + 2304\,e^2 + 6048\,e^4 + 3360\,e^6 + 315\,e^8)$ $/(\mathrm{sqrt}(1-e^2)\,(-128 + 1152\,e^2 - 4608\,e^4 + 10752\,e^6 - 16128\,e^8 + 16128\,e^{10}$ $- 10752\,e^{12} + 4608\,e^{14} - 1152\,e^{16} + 128\,e^{18}))$

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